GEOMETRIA DELLE AREE

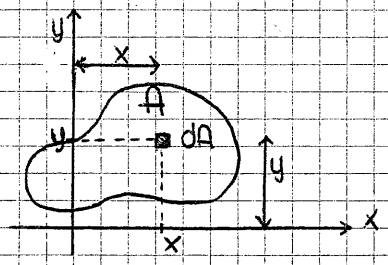
MOMENTI DEL 1° ORDINE o STATICI

$S_x = \int_A y\,dA$

$S_y = \int_A x\,dA$

$[S] = L^3$

$S \gtrless 0$



BARICENTRO:

$x_G = \frac{\int_A x\,dA}{A} = \frac{S_y}{A} \rightarrow S_y = A\,x_G$

$y_G = \frac{\int_A y\,dA}{A} = \frac{S_x}{A} \rightarrow S_x = A \cdot y_G$

PROPRIETÀ:

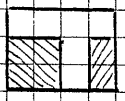
1) Il momento statico rispetto ad un asse baricentrico è = 0
2) Se una figura piana ha un asse di simmetria, il momento statico rispetto ad esso è = 0
   ↳ il baricentro si trova sull'asse di simmetria
3) Se una figura piana ha due assi di simmetria, la loro intersezione ≡ G
4) $S_x = \Sigma S_{xi}$; $S_y = \Sigma S_{yi}$ per la proprietà additiva degli integrali



→ somma di due rettangoli:



↳ differenza tra un rettangolo grosso e due piccoli:

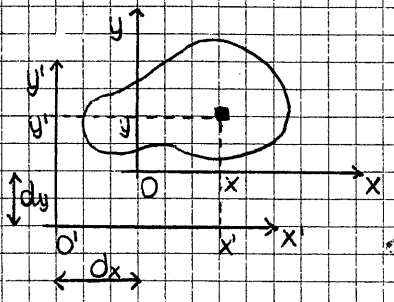


TRASLAZIONE DEGLI ASSI:

$\begin{cases} x' = x + d_x \\ y' = y + d_y \end{cases}$

$S_{x'} = \int_A y'\,dA = \int_A (y + d_y)\,dA = \int_A y\,dA + d_y \int_A dA = S_x + d_y \cdot A$

$S_{y'} = S_y + d_x \cdot A$

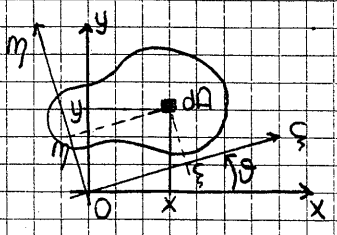


ROTAZIONE DEGLI ASSI:

$\begin{cases} \xi = x\cos\vartheta + y\,\mathrm{sen}\vartheta \\ \eta = y\cos\vartheta - x\,\mathrm{sen}\vartheta \end{cases}$

$S_\xi = \int_A \eta\,dA = \int_A (y\cos\vartheta - x\,\mathrm{sen}\vartheta)\,dA = S_x\cos\vartheta - S_y\,\mathrm{sen}\vartheta$

$S_\eta = S_y\cos\vartheta + S_x\,\mathrm{sen}\vartheta$



MOMENTI DEL 2° ORDINE

MOMENTI DI INERZIA:

$J_x = \int_A y^2 dA$

$J_y = \int_A x^2 dA$

$[J_x] = [J_y] = L^4$

$J_x, J_y > 0$

MOMENTO CENTRIFUGO:

$J_{xy} = \int_A x \cdot y \cdot dA$

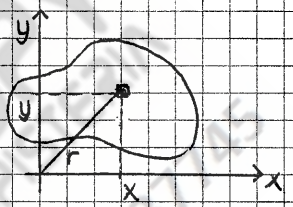
$[J_{xy}] = L^4$

$J_{xy} \gtrless 0$

Se uno dei due assi rispetto ai quali è valutato J_{xy} è di simmetria, allora $J_{xy} = 0$

MOMENTO D'INERZIA POLARE:

$J_O = \int_A r^2 dA$

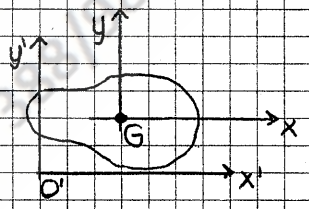


$[J_O] = L^4$

$J_O > 0$

$J_O = \int_A r^2 dA = \int_A (x^2 + y^2) dA = J_y + J_x$

TRASLAZIONE DEGLI ASSI:



$\begin{cases} x' = x + d_x \\ y' = y + d_y \end{cases}$

$J_{x'} = \int_A y'^2 dA = \int_A (y + d_y)^2 dA = \int_A (y^2 + 2y d_y + d_y^2) dA = \int_A y^2 dA + 2d_y \int_A y\, dA + d_y^2 \int_A dA =$

$= J_x + d_y^2 \cdot A$

$\int_A y\, dA = S_x = 0$

$J_{y'} = J_y + d_x^2 \cdot A$

$J_{x'y'} = \int_A x' y' dA = \int_A (x + d_x)(y + d_y) dA = \int_A xy\, dA + d_y \int_A x\, dA + d_x \int_A y\, dA + d_x d_y \int_A dA =$

$= J_{xy} + d_x d_y A$

ROTAZIONE DEGLI ASSI o FORMULE DI STEINER:

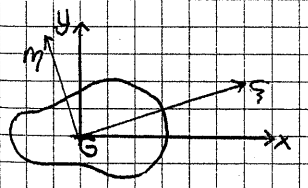
$J_\xi = \int_A \eta^2 dA = \int_A (y\cos\theta + x\,\mathrm{sen}\theta)^2 dA = \int_A (y^2\cos^2\theta + 2xy\,\mathrm{sen}\theta\cos\theta + x^2\,\mathrm{sen}^2\theta) dA =$

$= \cos^2\theta \int_A y^2 dA + 2\,\mathrm{sen}\theta\cos\theta \int_A xy\, dA + \mathrm{sen}^2\theta \int_A x^2 dA = J_x \cos^2\theta - 2J_{xy}\,\mathrm{sen}\theta\cos\theta + J_y \mathrm{sen}^2\theta$

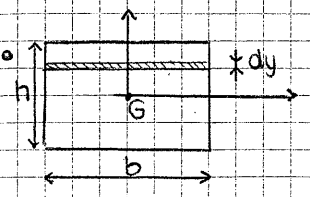
$J_\eta = J_x \mathrm{sen}^2\theta + 2J_{xy}\,\mathrm{sen}\theta\cos\theta + J_y \cos^2\theta$

$J_{\xi\eta} = J_{xy}(\cos^2\theta - \mathrm{sen}^2\theta) + (J_x - J_y)\,\mathrm{sen}\theta\cos\theta$

$J_O = J_x + J_y = J_\xi + J_\eta$ ← J_O non varia se ruoto gli assi



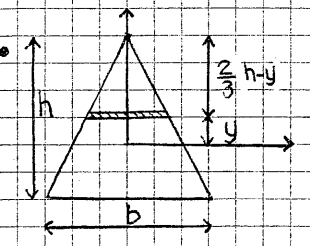
Esercizi:

- 

$J_x = \int_A y^2 dA = \int_{-h/2}^{h/2} b y^2 dy = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = b\left(\frac{h^3}{24} + \frac{h^3}{24}\right) = \frac{bh^3}{12}$

$dA = b\,dy$

$J_y = \frac{b^3 h}{12}$

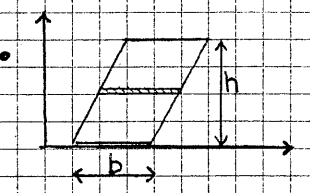
- 

$J_x = \int_A y^2 dA$

Sfrutto la similitudine dei triangoli:

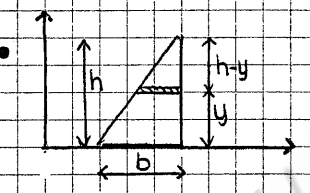
$b : h = b(y) : \frac{2}{3}h - y \quad ; \quad b(y) = \frac{b}{h}\left(\frac{2}{3}h - y\right) \qquad dA = b(y)\,dy$

$J_x = \int_{-h/3}^{2h/3} \frac{b}{h}\left(\frac{2}{3}h - y\right) y^2 dy = \left[\frac{2}{9}by^3 - \frac{b}{4h}y^4\right]_{-h/3}^{2h/3} = \frac{bh^3}{36}$

- 

$S_x = \int_A y\,dA = \int_0^h by\,dy = \frac{1}{2}bh^2$ è come quello del rettangolo

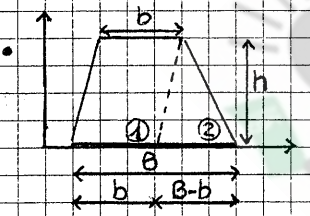
$dA = b\,dy$

- 

Sfrutto la similitudine dei triangoli:

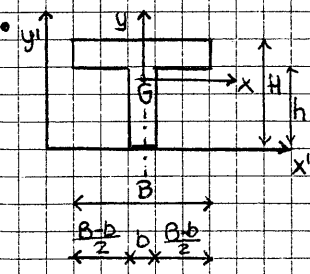
$b : h = b(y) : h - y \quad ; \quad b(y) = \frac{b}{h}(h - y) \qquad dA = b(y)\,dy$

$S_x = \int_A y\,dA = \int_0^h \frac{b}{h}(h-y)\,y\,dy = \left[\frac{by^2}{2} - \frac{by^3}{3h}\right]_0^h = \frac{bh^2}{2} - \frac{bh^2}{3} = \frac{bh^2}{6}$

- 

Trapezio = parallelogramma + triangolo

$S_x = S_x^{①} + S_x^{②} = \frac{1}{2}bh^2 + \frac{(B-b)h^2}{6}$

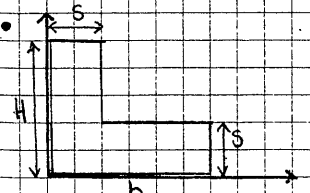
- 

- $S_x' = \frac{1}{2}BH^2 - \left(\frac{1}{2}\frac{B-b}{2}h^2\right)\cdot 2$

- $S_x' = \frac{1}{2}B(H-h)^2 + \frac{1}{2}bh^2$

Baricentro: $S_x' = S_x + y'_G \cdot A$ con $S_x = 0$

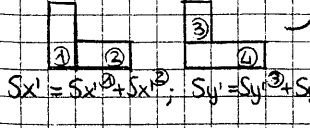
$y'_G = \frac{S_x'}{A} = \frac{BH^2 - (B-b)h^2}{2\,[BH - (B-b)h]}$

- 

$x'_G = \frac{S_{y'}}{A} \quad ; \quad y'_G = \frac{S_{x'}}{A}$

$A = s\cdot h + s\cdot b - s\cdot s = s(h+b-s)$

$\begin{cases} S_{x'} = \frac{1}{2}sh^2 + \frac{1}{2}(b-s)s^2 = \frac{1}{2}s(h^2 + bs - s^2) \rightarrow y'_G = \frac{S_{x'}}{A} = \frac{s(h^2+bs-s^2)}{2s(h+b-s)} \\ S_{y'} = \frac{1}{2}(h-s)s^2 + \frac{1}{2}sb^2 = \frac{1}{2}s(b^2 + hs - s^2) \rightarrow x'_G = \frac{S_{y'}}{A} = \frac{s(b^2+hs-s^2)}{2s(h+b-s)} \end{cases}$



$S_{x'} = S_{x'}^{①} + S_{x'}^{②} \quad ; \quad S_{y'} = S_{y'}^{③} + S_{y'}^{④}$

Se $s = b$ e $s = h$: $x'_G = \frac{b^2}{h+b} \quad ; \quad y'_G = \frac{h^2}{h+b}$

MOMENTI PRINCIPALI DI INERZIA

È possibile trovare un sistema di riferimento per cui $J_{\xi\eta} = 0$

$J_{\xi\eta} = 0$

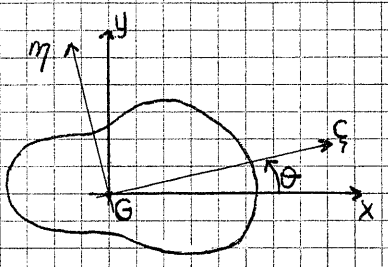
$J_{xy}(\cos^2\theta - \sin^2\theta) + (J_x - J_y)\sin\theta\cos\theta = 0$

$J_{xy}\cos 2\theta + \frac{1}{2}(J_x - J_y)\sin 2\theta = 0$

$J_{xy} + \frac{1}{2}(J_x - J_y)\,\mathrm{tg}\,2\theta = 0$

$\mathrm{tg}\,2\theta = -\frac{2J_{xy}}{J_x + J_y} = \frac{2J_{xy}}{J_y - J_x}$

$\theta = \mathrm{arctg}\left(\frac{2J_{xy}}{J_y - J_x}\right)$



θ mi identifica il SISTEMA DI RIFERIMENTO PRINCIPALE D'INERZIA

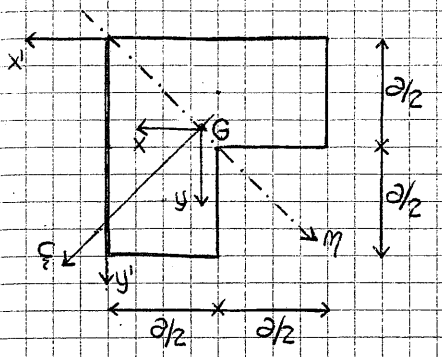
ξ e η sono ASSI PRINCIPALI D'INERZIA

(in questo caso sono anche baricentrici)

J_ξ e J_η sono MOMENTI PRINCIPALI D'INERZIA

www.mosaicocorsi.it 050/8531126 388/9837745

Esercizio:



Trovare $J_ç$ e J_η

η è asse di simmetria

η e ç sono assi principali baricentrici d'inerzia

formule di Steiner: $J_ç = J_x \cos^2\vartheta - 2J_{xy} \sin\vartheta\cos\vartheta + J_y \sin^2\vartheta$

$J_\eta = J_x \sin^2\vartheta + 2J_{xy} \sin\vartheta\cos\vartheta + J_y \cos^2\vartheta$

non conosco J_x, J_y e J_{xy}: $J_x = J_{x'} - y_G'^2 \cdot A$

$J_y = J_{y'} - x_G'^2 \cdot A$

J_{xy} lo calcolo con la formula

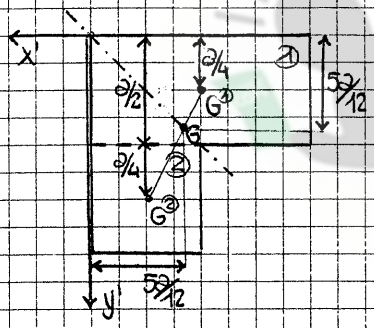
non conosco $J_{x'}$ e $J_{y'}$: $J_{x'}$ lo calcolo con la formula

$J_{y'}$ lo calcolo con la formula

non conosco x_G e y_G: $x_G' = \frac{S_{y'}}{A}$

$y_G' = \frac{S_{x'}}{A}$

$S_{x'}$, $S_{y'}$ e A li posso calcolare direttamente.



$A = A^{①} + A^{②} = a\frac{a}{2} + \frac{a}{2}\cdot\frac{a}{2} = \frac{3}{4}a^2$

$S_{x'} = S_{x'}^{①} + S_{x'}^{②} = \underbrace{a\cdot\frac{a}{2}}_{A^{①}} \cdot \underbrace{\frac{a}{4}}_{y_G'^{①}} + \underbrace{\frac{a}{2}\cdot\frac{a}{2}}_{A^{②}} \cdot \underbrace{\left(\frac{a}{2}+\frac{a}{4}\right)}_{y_G'^{②}} = \frac{5}{16}a^3$

$y_G' = \frac{S_{x'}}{A} = \frac{5}{16}a^3 \cdot \frac{4}{3a^2} = \frac{5}{12}a$

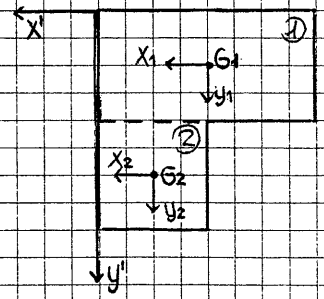
$x_G' = y_G'$ per simmetria rispetto a η

$J_{x'} = J_{x'}^{①} + J_{x'}^{②} = J_{X_1} + A^{①}\cdot y_{G1}^2 + J_{X_2} + A^{②}\cdot y_{G2}^2 =$

$= \frac{1}{12}a\left(\frac{a}{2}\right)^3 + a\cdot\frac{a}{2}\cdot\left(\frac{a}{4}\right)^2 + \frac{1}{12}\frac{a}{2}\left(\frac{a}{2}\right)^3 + \frac{a}{2}\frac{a}{2}\cdot\left(\frac{a}{4}+\frac{a}{2}\right)^2 =$

$= a^4\left(\frac{1}{12\cdot 8} + \frac{1}{32} + \frac{1}{24\cdot 8} + \frac{9}{4\cdot 16}\right) = a^4\,\frac{2+6+1+27}{2^6\cdot 3} = a^4\,\frac{36}{2^6\cdot 3}$

$= \frac{3}{16}a^4$



$J_{y'} = J_{x'}$ per simmetria rispetto a η

per calcolare $J_{y'}$ devo dividere la figura in questo modo →

